

# Transmission Code Design for Asynchronous Full-Duplex Relaying

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**Abstract** - Full-duplex relay systems have emerged as a promising solution to improve spectrum efficiency. Due to residual self-interference, space-time code may lose diversity gain. This study addresses this problem caused by self-interference and asynchronous transmission in full-duplex relay systems. In particular, a space-time code is designed at the transmitter and the relay to achieve diversity gain at the presence of residual self-interference. During transmission, the code at the relay can be used to estimate the residual self-interference caused by imperfect channel knowledge. The estimation result can be used to further reduce self-interference. Diversity gain can be achieved at the destination as a result of the reduction of the residual self-interference. Both theoretical analysis and simulation results demonstrate the diversity gain of our design in asynchronous full-duplex relay systems.

**Keywords:** Full-duplex relay, asynchronous transmission, self-interference, space-time code.

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## 1. Introduction

The full-duplex transmission mode has been investigated to provide high data rate and diversity gain in relay systems [1–6]. Full-duplex relay systems can theoretically double the spectrum efficiency of traditional half-duplex relay systems. Moreover, a full-duplex node can sense other transmissions during its transmitting, to avoid the hidden node problem [7].

A major hindrance encountered in full-duplex relay systems is self-interference because signal transmission and reception are simultaneously performed at the same relay in a full-duplex mode. Worse still, self-interference may be much stronger than the signal of interest. Many efforts have been exerted at cancelling the self-interference [8–12], and current self-interference cancellation can be achieved in the range between 66 and 74 dB [13]. However, as discussed in [13], given the large residual self-interference (RSI), the 66 dB cancellation is insufficient for full-duplex relay systems to outperform half-duplex relay systems in terms of data rate. Furthermore, the signal cancellation techniques require a bulky radio-frequency attenuator, which is expensive to implement [7]. Moreover, the high RSI in full-duplex relay systems causes a high bit error, which presents diversity-zero performance; that is, error floor exists in a high signal-

to-noise ratio (SNR) region [14]. To alleviate this problem, [15, 16] designed optimal power allocation.

Asynchronous transmission is another hindrance encountered in full-duplex relay systems in practice. Two transmission links exist in a typical full-duplex relay system: a direct link from the source to the destination and a relay link that connects the source and the destination. The direct link contains the path delay from the source to the destination, and the relay link contains both the processing delay at the relay and transmission delay. The delay difference between the relay and the direct links results in asynchronous transmission. Such asynchronous transmission may cause two problems: a loss in diversity and an increase in decoding complexity. For an ideal full-duplex relay systems, the space-time code (STC) can be adopted at the source and relays to harvest diversity gain [17, 18]. However, the code structure of STC may not be full rank because of asynchronous transmission, thereby resulting in diversity loss [17, 18]. For maximum-likelihood (ML) decoding, the decoding complexity may increase exponentially with code length and linearly with delay difference.

The work in [19] considered a full-duplex system with no path delay. The proposed code design in [19] added an intentional delay on each relay to form a full diversity code structure, thereby ensuring full diversity. However, such code structure may confront diversity loss when considering path delay and processing delay at relays. In [18], the code design deploys a distributed linear convolutive code to ensure full diversity. However, the code design requires perfect knowledge of the self-interference channel for encoding, and its decoding is complexity prohibitive when the code length is long.

In the present work, we jointly consider the effects of RSI and delays. The code structure in our code design is specifically separated into two parts. The first part is used to estimate the RSI at the relay. With the estimation result, the relay can cancel the RSI when transmitting the second part. Our design improves the bit error rate (BER) performance in asynchronous full-duplex relay systems.

## 2. Asynchronous Full-Duplex Relay System

We consider an asynchronous full-duplex relay system as shown in Figure 1, in which the relay R helps the transmission between source S and destination D. A direct transmission link exists between source S and destination D. The direct and relay links form a  $2 \times 1$

system with potential of diversity 2. The channel fading and delay coefficient of the direct link are  $h_{SD}$  and  $\tau_{SD}$ , respectively; those between the source and the relay are  $h_{SR}$  and  $\tau_{SR}$ , respectively; and those between the relay and the destination are  $h_{RD}$  and  $\tau_{RD}$ , respectively. The channel fading coefficients  $h_{SD}$ ,  $h_{SR}$  and  $h_{RD}$  follow complex Gaussian distribution  $\mathcal{CN}(0, 1)$ , and remain unchanged during the transmission of one code block. The transmission power of the source is  $P_T$ . The delay coefficients are known at each node as in [17, 18]. The delay difference is given as

$$\tau_1 = \tau_{SR} + \tau_{RD} - \tau_{SD} \geq 0 \quad (1)$$

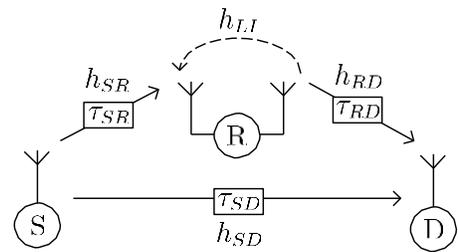


Figure 1. An asynchronous full-duplex relay system with one relay and direct link.

### 2.1. Residual Self-Interference

The relay works in full-duplex mode, which means that the relay simultaneously receives and transmits signals within the same frequency. The loop channel between the transmit and receive antennas of the relay is denoted as  $h_{LI}$ , which remains unchanged during the transmission of one code block [13]. At the relay, the estimation of  $h_{LI}$  is  $\hat{h}_{LI}$ . Theoretically, if  $\hat{h}_{LI} = h_{LI}$ , then we can perfectly cancel the self-interference signal. However, the estimation of  $h_{LI}$  is not perfect in practice. The estimation error is defined as  $\delta \triangleq \hat{h}_{LI} - h_{LI}$ , where  $\delta$  is modeled as a complex random variable that varies from each code block to another, but remains unchanged in the same code block. Many efforts have been exerted at improving the self-interference cancellation [8–13]. For a typical radio transmission, the SNR is approximately 30 dB. The self-interference can be millions of times stronger (60 dB or more) than a received signal. In such case, nearly 100 dB cancellation of self-interference is required to ensure the 30 dB gap in the signal and interference plus noise. Currently, many schemes [8–13] that use analog cancellation, digital cancellation, antenna separation, or some combination of the above are able to cancel the self-interference around 74 dB [9, 13], such that the

power of RSI can be as low as  $-10\text{dB}$  compared with the signal of interest.

## 2.2. Transmission Model

At the beginning, the source  $S$  transmits a sequence  $\mathbf{s}[1:l]$ , where  $l \in \mathbb{Z}^+$  is the length of the sequence. In the amplifying-and-forward mode, the received signal  $r[i]$  and the transmitted signal  $t[i]$  at the relay  $R$  at time  $i$  are expressed as follows:

$$r[i] = \begin{cases} n_R[i], & i \leq \tau_{SR} \\ \sqrt{P_T} h_{SR} s[i - \tau_{SR}] + n_R[i], & i = \tau_{SR} + 1 \\ h_{LI} t[i] + n_R[i], & i = \tau_{SR} + l + 1 \\ \underbrace{\sqrt{P_T} h_{SR} s[i - \tau_{SR}]}_{\text{signal}} + \underbrace{h_{LI} t[i]}_{\text{self-interference}} + \underbrace{n_R[i]}_{\text{noise}}, & \text{others} \end{cases} \quad (1)$$

$$t[i] = \begin{cases} 0, & 1 \leq i \leq \tau_{SR} + 1 \\ \alpha(r[i - 1]), & \tau_{SR} + 2 \leq i \leq \tau_{SR} + l + 1 \end{cases}$$

where  $P_T$  is the transmission power at the source  $S$ ,  $\alpha$  is the amplifying factor at the relay  $R$ , and  $n_R[i]$  that follows  $\mathcal{CN}(0, \sigma^2)$  is the noise at the relay  $R$ ,  $i = 1, 2, \dots, \tau_{SR} + l + 1$ . We consider that the relay can use  $\hat{h}_{LI}$  to cancel the self-interference [8–13]. After self-interference cancellation, the received signal becomes

$$\hat{r}[i] = \begin{cases} \sqrt{P_T} h_{SR} s[i - \tau_{SR}] + \delta t[i] + n_R[i], & \\ \tau_{SR} + 2 \leq i \leq \tau_{SR} + l + 1 \end{cases} \quad (2)$$

where  $\delta t[i]$  is a RSI term and  $\delta = \hat{h}_{LI} - h_{LI}$  is a random variable. Then, the transmitted signal  $t[i]$  based on  $\hat{r}[i]$  ( $\tau_{SR} + 2 \leq i \leq \tau_{SR} + l + 1$ ) is given as

$$t[i] = \alpha \sqrt{P_T} h_{SR} s[i - \tau_{SR} - 1] + \alpha \delta t[i - 1] + n_R[i - 1] \quad (3)$$

The received signal vector after self-interference cancellation is

$$\begin{bmatrix} \hat{r}[1: \tau_{SR} + l + 1] \\ 0_{\tau_{SR}} s[1] \cdots s[l] 0 \\ \vdots \quad \vdots \quad \ddots \quad \vdots \quad \vdots \\ 0_{\tau_{SR}} 0 \cdots 0 s[1] \end{bmatrix} + [1, \alpha \delta, \dots, \alpha^{l-1} \delta^{l-1}, \alpha^l \delta^l] \quad (4)$$

$$\begin{bmatrix} n_R[1: \tau_{SR} + 1] & n_R[\tau_{SR} + 1] \cdots & n_R[\tau_{SR} + 1] & n_R[\tau_{SR} + l + 1] \\ \vdots & \ddots & \vdots & \vdots \\ 0_{\tau_{SR}} & 0 \cdots 0 & 0 & n_R[\tau_{SR} + 1] \end{bmatrix}$$

The coefficients  $[\alpha \delta, \dots, \alpha^l \delta^l]$  are caused by RSI terms, where  $\mathbb{E}[|\alpha \delta^2|]$  is typically  $-10\text{ dB}$ , as discussed

in Section 2.1. Without loss of generality, we set the fixed-gain coefficient  $\alpha = 1$ , and model  $\delta$  at  $\mathcal{CN}(0, \sigma^2)$  with  $\sigma^2 = 0.1$ .

## 2.3. Performance Loss from Residual Self-Interference

Since the self-interference channel  $h_{LI}$  is much stronger than other channel coefficients, the estimation accuracy of  $h_{LI}$  is very important to full-duplex relay networks. The existence of channel estimation error would cause severe bit error rate (BER) degradation in delay diversity code [22] as shown by our simulation in Figure 2. If the power of RSI is  $-10\text{dB}$  or  $-15\text{dB}$ , the BER simulation would encounter error floor around  $10^{-3}$  or  $10^{-5}$ , respectively. From the state of the art, the power of RSI is around  $-10\text{dB}$  [9, 13]. For cases with RSI as  $-20\text{dB}$  and  $-30\text{dB}$ , the performance degradation is not so severe when  $P_T/N_0$  is lower than  $40\text{dB}$ . In other words, for practical applications, if we manage to reduce the RSI to  $-20\text{dB}$  or below, the effect of RSI may not be so significant.

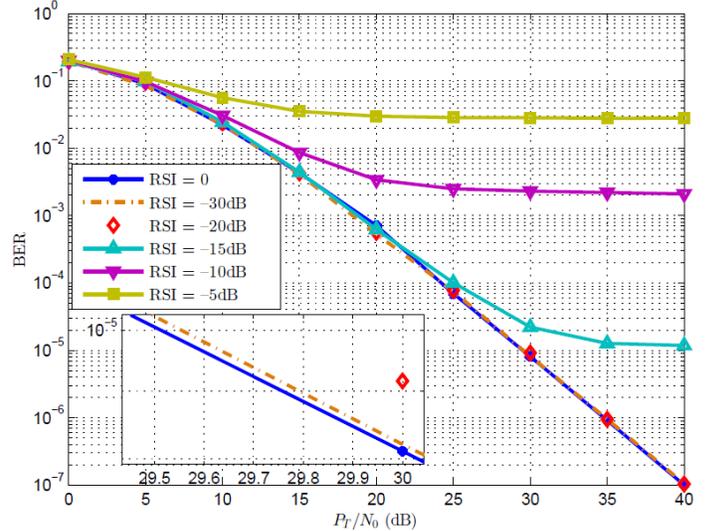


Figure 2. Bit error rate performance of delay diversity code [22] when RSI is  $\{0, -30\text{dB}, -20\text{dB}, -15\text{dB}, -10\text{dB}, -5\text{dB}\}$  using code length 8 and  $\alpha = 1$ .

## 3. Estimation of Residual Self-Interference

In the above description, we focus on the transmitted and received signals at relay  $R$ . Specifically, source  $S$  transmits  $l$  symbols and remains silent in the next one symbol duration in each transmission block, where channel coefficients remain unchanged. Accurate estimation of  $h_{LI}$  is very important to cancel the self-interference at the relay. However, the estimation error

$\delta$  exists in practice. To cancel the RSI, it is important to accurately estimate  $\delta$  on the relay. However, the received signal vector in (4) contains  $s[1:l]$ , which is unknown to the relay.

In this section, we propose a method of joint estimation and detection. After retransmitting the  $l$  symbols, the received signal vector at relay R is given in (4). Term  $\delta$  appears repeatedly  $l$  times in  $\hat{r}[\tau_{SR} + 2:\tau_{SR} + l + 1]$ . Thus, all these signals can be used to estimate  $\delta$ . In this section, we show how to estimate the RSI at relay R. We first transform (4) into the following linear model:

$$\begin{bmatrix} \hat{r}[\tau_{SR} + 2] \\ \vdots \\ \hat{r}[\tau_{SR} + l] \\ \hat{r}[\tau_{SR} + l + 1] \end{bmatrix} = \begin{bmatrix} \hat{r}[\tau_{SR} + 1] \\ \vdots \\ \hat{r}[\tau_{SR} + l - 1] \\ \hat{r}[\tau_{SR} + l] \end{bmatrix} \delta + \sqrt{P_T} h_{SR} \begin{bmatrix} s[2] \\ \vdots \\ s[l] \\ 0 \end{bmatrix} + \begin{bmatrix} n_R[\tau_{SR} + 2] \\ \vdots \\ n_R[\tau_{SR} + l] \\ n_R[\tau_{SR} + l + 1] \end{bmatrix} \quad (5)$$

In this equation, the relay knows the received  $(l + 1)$ -length signal sequence  $\hat{r}[\tau_{SR} + 1:\tau_{SR} + l + 1]$ , transmission power  $P_T$  and channel coefficient  $h_{SR}$ , and the relay aims to derive the channel estimation error  $\delta$  and signal  $(l-1)$ -length sequence  $s[2:l]$ . In this section, we propose estimation methods at the relay. The estimation results are denoted as  $(\hat{\delta}, \hat{s}[2:l])$ . Ideally, perfect channel estimation leads to  $\hat{\delta} = \delta$ , and perfect signal detection lead to  $\hat{s}[2:l] = s[2:l]$ .

From (5), we can derive the estimation of  $\delta$  in (6).

$$\hat{\delta} = \frac{1}{\sum_{i=\tau_{SR}+1}^{\tau_{SR}+l} |\hat{r}[i]|^2} \left[ \hat{r}^*[\tau_{SR} + 1], \dots, \hat{r}^*[\tau_{SR} + l - 1], \hat{r}^*[\tau_{SR} + l] \right] \begin{bmatrix} \hat{r}[\tau_{SR} + 2] - \sqrt{P_T} h_{SR} \hat{s}[2] \\ \vdots \\ \hat{r}[\tau_{SR} + l] - \sqrt{P_T} h_{SR} \hat{s}[l] \\ \hat{r}[\tau_{SR} + l + 1] \end{bmatrix} \quad (6)$$

The processes of RSI estimation and signal detection essentially involve finding the  $(\hat{\delta}, \hat{s}[2:l])$  that maximizes the logarithm probability in (7).

$$\ln p(\hat{r}[\tau_{SR} + 2:\tau_{SR} + l + 1] | \hat{\delta}, \hat{s}[2:l]) = -\ln(\pi\sigma^2) - \frac{1}{\sigma^2} \left( |\hat{r}[\tau_{SR} + l + 1] - \hat{\delta} \hat{r}[\tau_{SR} + l]|^2 + \sum_{i=1}^{l-1} |\hat{r}[i + \tau_{SR} + 1] - \hat{\delta} \hat{r}[i + \tau_{SR}] - \sqrt{P_T} h_{SR} \hat{s}[i + 1]|^2 \right) \quad (7)$$

The basic idea is to use all possible combinations of  $\hat{s}[2:l]$  to estimate  $\delta$  in (6), and then calculate

$\ln p(\hat{r}[\tau_{SR} + 2:\tau_{SR} + l + 1] | \hat{\delta}, \hat{s}[2:l])$  to find the maximum one. One simple approach is to calculate (7) using all possible candidates of  $\hat{s}[2:l]$  to obtain the values of  $\hat{\delta}$  from the maximum one. When  $\hat{s} = s$ , the modified Cramér-Rao lower bound (MCRLB) [21] for estimating  $\delta$  is in (8) is

$$\text{MCRLB} = \mathbb{E} \left\{ \frac{\sigma^2}{\sum_{i=\tau_{SR}+1}^{\tau_{SR}+l} |\hat{r}[i]|^2} \right\} \quad (8)$$

When SNR is significantly high, the detection is likely to be error-free, and the estimated  $\delta$  may not be affected by detection errors. At the relay, the complexity of this estimation scheme is  $\mathcal{O}(|\mathcal{S}|^{l-1})$ .

#### 4. Design of Space-Time Code

In this section, we propose for the asynchronous full-duplex relay system an STC that estimates  $\delta$  first to cancel the RSI during the relay transmission at the relay. We also analyze the diversity gain, code rate, and decoding complexity of our design.

We assume that source S intends to transmit an  $L$ -symbol sequence  $\tilde{s}$  to the destination in a full-duplex relay system, as shown in Figure 1. The transmitted signal vector by source S is defined as:

$$\tilde{s} = \sqrt{P_T} \begin{bmatrix} \underbrace{s[1], s[2], \dots, s[l]}_{\text{transmission part I}}, & \underbrace{0_{\tau_1+1}}_{\text{zero-padding part}}, & \underbrace{s[l+1], \dots, s[L]}_{\text{transmission part II}} \end{bmatrix} \quad (9)$$

where the first part of the transmission is used to estimate the RSI at the relay. With the estimation result, relay R can cancel the RSI when sending the second part. The zero-padding part is inserted after transmitting  $l$  ( $1 \leq l \leq L$ ) symbols to separate the two transmission parts, in which  $\tau_1$  is the delay deference defined in (1).

The relay receives and transmits signals in the following procedure. During the first  $\tau_{SR} + l + 1$  time slots, the relay jointly estimates  $\hat{\delta}$  and detects  $s[2:l]$  to obtain  $\hat{\delta}$  by applying scheme in Section 3. Meanwhile, during time slots  $[\tau_{SR} + 2:\tau_{SR} + l + 1]$ , the relay directly transmits the received signal in the previous time slot. After reducing the RSI by  $\hat{\delta}$ , the relay transmits the processed signals in the time slots  $[\tau_{SR} + l + \tau_1 + 3:\tau_{SR} + L + \tau_1 + 2]$ . The received signal vector at the relay is

$$\hat{\mathbf{r}} = \left[ \hat{\mathbf{r}}[1:\tau_{\text{SR}} + l + 1], 0_{\tau_1}, \hat{\mathbf{r}}[\tau_{\text{SR}} + l + \tau_1 + 2 : \tau_{\text{SR}} + L + \tau_1 + 2] \right],$$

where

$$\begin{aligned} & \hat{\mathbf{r}}[\tau_{\text{SR}} + l + \tau_1 + 2 : \tau_{\text{SR}} + L + \tau_1 + 2] \\ &= \sqrt{P_T} h_{\text{SR}} \left[ 1, \alpha(\delta - \hat{\delta}), \dots, \alpha^{L-l-1}(\delta - \hat{\delta})^{L-l-1}, \alpha^{L-l}(\delta - \hat{\delta})^{L-l-1} \right] \\ & \begin{bmatrix} s[l+1] & s[l+2] & \dots & s[L] & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & s[l+1] \end{bmatrix} \\ & + \left[ 1, \alpha(\delta - \hat{\delta}), \dots, \alpha^{L-l-1}(\delta - \hat{\delta})^{L-l-1}, \alpha^{L-l}(\delta - \hat{\delta})^{L-l-1} \right] \\ & \begin{bmatrix} n_{\text{R}}[\tau_{\text{SR}} + l + \tau_1 + 2] & n_{\text{R}}[\tau_{\text{SR}} + l + \tau_1 + 3] & \dots & n_{\text{R}}[\tau_{\text{SR}} + L + \tau_1 + 2] \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & n_{\text{R}}[\tau_{\text{SR}} + l + \tau_1 + 2] \end{bmatrix} \end{aligned}$$

can be derived from equation (2). The transmitted signal vector at the relay is

$$\mathbf{t} = \left[ \mathbf{t}[1:\tau_{\text{SR}} + l + 1], 0_{\tau_1+1}, \mathbf{t}[\tau_{\text{SR}} + l + \tau_1 + 3 : \tau_{\text{SR}} + L + \tau_1 + 2] \right],$$

where

$$\begin{aligned} & \mathbf{t}[\tau_{\text{SR}} + l + \tau_1 + 3 : \tau_{\text{SR}} + L + \tau_1 + 2] = \\ & \sqrt{P_T} h_{\text{SR}} \left[ \alpha, \alpha^2(\delta - \hat{\delta}), \dots, \alpha^{L-l}(\delta - \hat{\delta})^{L-l-1} \right] \\ & \begin{bmatrix} s[l+1] & s[l+2] & \dots & s[L] \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & s[l+1] \end{bmatrix} + \\ & \left[ \alpha, \alpha^2(\delta - \hat{\delta}), \dots, \alpha^{L-l}(\delta - \hat{\delta})^{L-l-1} \right] \\ & \begin{bmatrix} n_{\text{R}}[\tau_{\text{SR}} + l + \tau_1 + 2] & n_{\text{R}}[\tau_{\text{SR}} + l + \tau_1 + 3] & \dots & n_{\text{R}}[\tau_{\text{SR}} + L + \tau_1 + 1] \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & n_{\text{R}}[\tau_{\text{SR}} + l + \tau_1 + 2] \end{bmatrix} \end{aligned}$$

can be derived from equation (3). The transmission schemes in the transmitter and relay form an STC structure. At destination D, the received signal vector is

$$\mathbf{y}_D = \mathbf{n}_D + \sqrt{P_T} [h_{\text{SD}}, h_{\text{SR}} h_{\text{RD}}] \begin{bmatrix} 0_{\tau_{\text{SD}}} & s[1:l] & 0_{\tau_1+1} & s[l+1:L] & 0_{\tau_1+1} \\ 0_{\tau_{\text{RD}}} & \mathbf{t}[1:\tau_{\text{SR}} + l + 1] & 0_{\tau_1+1} & \mathbf{t}[\tau_{\text{SR}} + l + \tau_1 + 3 : \tau_{\text{SR}} + L + \tau_1 + 2] \end{bmatrix} \quad (10)$$

where  $\mathbf{n}_D$  is the noise vector at destination D, the channel coefficients are known. To illustrate the code structure, we give the following example.

*Example:* We let  $[\tau_{\text{SD}}, \tau_{\text{SR}}, \tau_{\text{RD}}] = [0, 0, 0]$ ,  $\tau_1 = 0$ ,  $l = 2$ , and  $L = 5$ . For simplicity, we let  $\alpha = 1$  and  $\hat{\delta} = \delta$ .

Equation (10) becomes

$$\mathbf{y}_D = \mathbf{n}_D + \sqrt{P_T} [h_{\text{SD}}, h_{\text{SR}} h_{\text{RD}}] \begin{bmatrix} s[1] & s[2] & 0 & s[3] & s[4] & s[5] & 0 \\ 0 & s[1] & s[2] + \delta s[1] & 0 & s[3] & s[4] & s[5] \end{bmatrix} \quad (11)$$

Equation (11) shows that the first  $l$  symbols, that is,  $s[1]$  and  $s[2]$ , do not interfere with the remaining  $L - l$  symbols, that is,  $s[3]$ ,  $s[4]$  and  $s[5]$ , because of the zero-padding  $0_{\tau_1+1}$  in (9). The decoding methods at the destination are different for the first  $l$  symbols and the remaining  $L - l$  symbols. An RSI exists for the first  $l$  symbols. We assume that the destination can obtain the resulting  $\hat{\delta}$  from the relay so that the destination can conduct exhaustive search on the  $l$  symbols. In practice, even if the destination cannot perfectly obtain  $\hat{\delta}$  from the relay, it can still detect the signal and similarly estimate the RSI as the relay does in Section 3, thereby resulting in the same exhaustive search on the  $l$  symbols. A remaining RSI exists for the other  $L - l$  symbols because  $\hat{\delta} \neq \delta$ . However, given that this remaining RSI after cancellation is sufficiently small, we can treat it as noise in the decoding process for the rest  $L - l$  symbols.

**Remark 1.** (Diversity Gain) In an asynchronous full-duplex relay system with one relay, one single-antenna source, and one single-antenna destination, the proposed STC achieves full diversity, that is, diversity order 2, with the perfect estimation of the RSI. This occurs because the reduction of self-interference makes the direct and relay links form an ideal  $2 \times 1$  system.

**Remark 2.** (Code Rate Analysis) The code rate of the proposed STC is  $\frac{L}{L + \tau_{\text{SD}} + 2\tau_1 + 2}$ . When  $L$  is sufficiently large, this rate approaches 1 symbol/channel.

**Remark 3.** (Decoding Complexity) The decoding complexity of the proposed STC at the destination is  $O(|\mathcal{S}|^l + (L - l) |\mathcal{S}|^2)$ . For the first  $l$  symbols, the destination obtains the parameter  $\hat{\delta}$  from the relay and exhaustively search over all possible  $s[1:l]$  with complexity  $\mathcal{O}(|\mathcal{S}|^l)$ . For the remaining  $L - l$  symbols, the relay reduces the RSI, and the remaining RSI is treated as noise at the destination. The Viterbi decoder, an equivalent form of ML decoder, is applied to decode

$s[l+1 : L]$  with complexity  $\mathcal{O}((L - l)|\mathcal{S}|^2)$ .

In practice, the decoding complexity is low when  $l$  is small and a simple constellation, such as QPSK is adopted. However, the estimation error variance is large when  $l$  is small, according to (8). If high-order constellations, such as 16QAM and 64QAM are adopted, then a large  $|\mathcal{S}|$  leads to high complexity.

## 5. Simulations

In this section, we simulate the mean square error (MSE) of RSI estimation and BER performance of the proposed STC with respect to SNR  $P_T/N_0$ , where  $N_0$  is the noise power spectrum density. The amplifying factor  $\alpha = 1$ , and RSI = -10 dB, as described in Section 2.1. The adopted constellation adopted is QPSK.

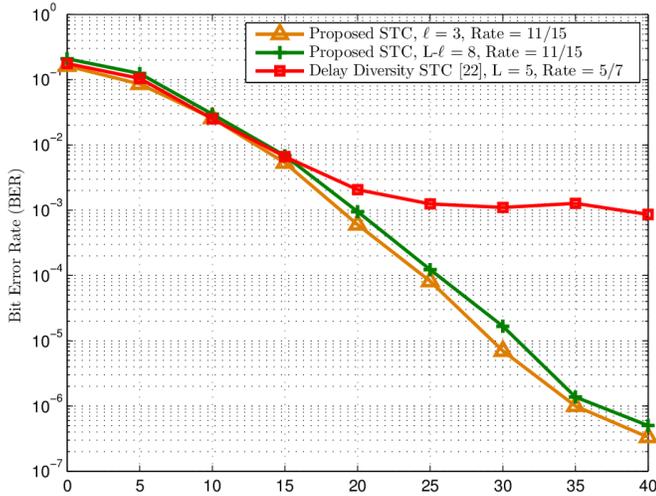


Figure 3. Comparison of the BER performance of the proposed STC and delay diversity STC [22] with respect to SNR.

Figure 3 compares the performance of the proposed STC and delay diversity STC [22] when  $\tau_1 = 1$ . For the sake of fairness, all the schemes use an ML decoder at the destination. The BER performance of delay diversity STC exhibits the error floor problem around BER  $10^{-3}$ . This problem is attributed to the existence of the RSI. By contrast, our proposed STC avoids such problems in the considered high  $P_T/N_0$  region. For the proposed STC, the BER performance of the  $l$  and  $L-l$  parts, which use different decoding methods as discussed in Section 4, are plotted separately. Both parts of the proposed STC demonstrate

diversity gain and therefore outperform delay diversity STC in terms of BER performance.

Figure 4 presents the estimation accuracy of the RSI on the relay with respect to a different SNR using different symbol length  $l$ . The MSE is defined as  $\mathbb{E}[|\delta - \hat{\delta}|^2]$ , which is the remaining RSI after estimation and cancellation. As the figure shows, the estimation can significantly reduce the RSI in the high SNR region. Moreover, the variance of the estimation error gradually approaches the MCRLB in the high SNR region. Moreover, the variance of the estimation error gradually approaches the MCRLB in the high SNR region for a given  $l$ , given that the variance of estimation error with perfect symbol knowledge can be regarded as the lower bound of the estimation accuracy, and the estimation accuracy improves in high SNR. In addition, MSE using  $l = 5$  is much smaller than that using  $l = 3$ . This observation verifies that an increase in symbol length  $l$  improves the estimation accuracy.

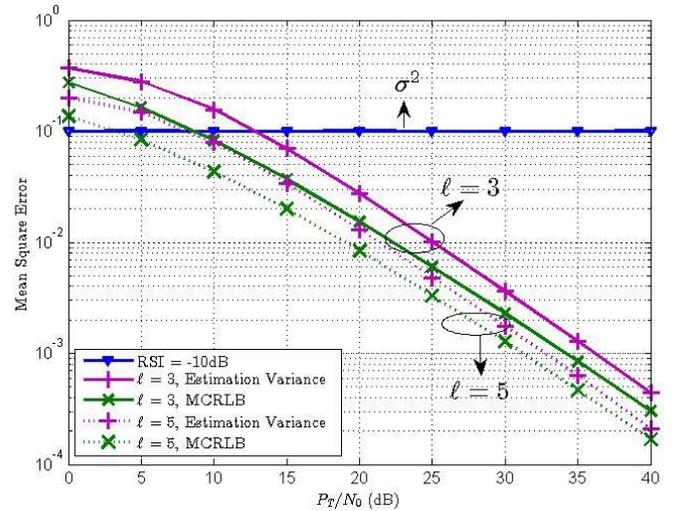


Figure 4. MSE of estimating  $\delta$  at the relay with respect to SNR using different  $l$ .

## 6. Conclusion

In this work, we considered a system with one full-duplex relay that assists the transmission between a single-antenna source and a single-antenna destination. We proposed an STC that estimates the RSI at the relay to achieve diversity gain at the destination. The simulations verified the BER performance with diversity gain of our proposed design.

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